A Tabu Search Heuristic with Efficient Diversification Strategies for the Class/Teacher Timetabling Problem

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The Class/Teacher Timetabling Problem (CTTP) deals with the weekly scheduling of encounters between teachers and classes of an educational institution. Since CTTP is a NP-hard problem for nearly all of its variants, the use of heuristic methods for its resolution is justified. This paper presents an efficient Tabu Search (TS) heuristic with two different memory based diversification strategies for CTTP. Results obtained through an application of the method to a set of real world problems show that it produces better solutions than a previously proposed TS found in the literature and faster times are observed in the production of good quality solutions.

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1. INTRODUCTION

The Class/Teacher Timetabling Problem (CTTP) embraces the scheduling of sequential encounters between teachers and students so as to insure that requirements and constraints are satisfied. Typically, the manual solution of this problem extends for various days or weeks and normally produces unsatisfactory results due to the fact that lesson periods could be scheduled which are inconsistent with pedagogical needs or could even serve as impediments for certain teachers or students. CTTP, in its optimization version, is a NP-hard problem [Even et al. 1976] for nearly all of its variants, justifying the usage of heuristic methods for its resolution. Therefore, various heuristic and metaheuristic approaches have been applied with success in the solution of this problem, such as: Tabu Search (TS) [Souza et al. 2003; Costa 1994; Schaerf 1999], Genetic Algorithms [Wilke et al. 2002] and Simulated Annealing (SA) [Abramson 1991].

The application of TS to the CTTP is specially interesting, since this method is, as local search methods generally are, very well suited for the interactive building of timetables. Furthermore, TS based algorithms offer robust solution methods for timetabling problems [Dowsland 1997], often presenting the best known solutions, when compared to other metaheuristics [Colorni et al. 1998; Souza 2000]. The diversification strategy is an important aspect in the design of a TS algorithm. Since the use of a tabu list is not enough to prevent the search process from becoming trapped in certain regions of the search space, other mechanisms have been proposed. In particular, for the CTTP, two main approaches have been used: adaptive relaxation [Schaerf 1999; Costa 1994] and random restart [Souza et al. 2003]. In adaptive relaxation the costs involved in the objective function are dynamically changed to guide the search process to newly, unvisited, regions of the search space. In random restart a new solution is generated and no previous information is used.

This work employs a TS algorithm that uses informed diversification strategies, which take into account the history of the search process to guide the selection of diversification movements. Successful implementations of these ideas can be found in [Gendreau et al. 1994; Sun 2006]. Although it uses only standard TS components, it provides better results than more complex previous proposals [Souza et al. 2003].

The article is organized as follows: section 2 presents related works; section 3 introduces the problem to be treated; section 4 presents the proposed algorithm; section 5 describes the computational experiments and their results; and finally, section 6 formulates conclusions and future research proposals.

2. RELATED WORKS

Although the CTTP is a classical combinatorial optimization problem, no widely accepted model is used in the literature. The reason is that the characteristics of the problem are highly dependent on the educational system of the country and the type of institution involved. As such, although the basic search problem is the same, variations are introduced in different works (mainly in the evaluation of timetables) [Colorni et al. 1998; Costa 1994; Schaerf 1999; Souza et al. 2003]. Described afterwards, the problem considered in this paper derives from [Souza et al. 2003] and considers the timetabling problem encountered in typical Brazilian high schools. In [Souza et al. 2003], a GRASP-Tabu Search (`GTS-II`) metaheuristic was developed to tackle this problem. The `GTS-II` method incorporates a specialized improvement procedure named "Intraclasses-Interclasses", which uses a shortest-path graph algorithm. At first, the procedure is activated aiming to attain the feasibility of the constructed solution, after which, it then aims to improve the feasible solution. The movements made in the "Intraclasses-Interclasses" also remain with tabu status for a given number of iterations. Diversification is implemented through the generation of new solutions, in the GRASP constructive phase. In [Souza 2000] three different metaheuristics that incorporate the "Intraclasses-Interclasses" were proposed: Simulated Annealing, Microcanonical Optimization (MO) and Tabu Search. The TS proposal significantly outperformed both SA and MO.

3. THE PROBLEM CONSIDERED

The problem considered deals with the scheduling of encounters with teachers and classes over a weekly period. The schedule is made up of d days of the week with h daily periods, defining the set P, with $p = d \times h$ distinct periods. There is a set T with t teachers which teach to a set C of c classes, which are disjoint sets of students with the same curriculum. The allocation of teachers to classes is previously fixed and the workload is given in a matrix of requirements $R_{t \times c}$, where r_{ij} indicates the number of lessons that teacher i shall teach for class j. Classes are available at

Teacher \ Period	1	2	3	4	5	$\cdots d \times h$
1	1	0	0	2	2	$\cdots$
2	0	X	X	0	1	$\cdots$
3	X	X	1	0	3	$\cdots$
4	0	1	0	1	0	$\cdots$
5	0	0	2	3	X	$\cdots$

Fig. 1. Fragment of generated timetable

any period, and must have their time schedules, of length p, completely filled out, while each teacher i indicates his/her set of available periods A_i. Also, teachers may request a number of double lessons per class. These lessons must be allocated in two consecutive periods on the same day. This way a solution to the CTTP problem must satisfy the following constraints:

(a) no class or teacher can be allocated for two lessons in the same period;
(b) teachers can only be allocated respecting their availabilities;
(c) each teacher must fulfill his/her weekly number of lessons;
(d) for pedagogical reasons no class can have more than two lesson periods with the same teacher per day.

Also, there are the following desirable features that a timetable should present:

(a) the time schedule for each teacher should encompass the least possible number of days;
(b) double lessons requests must be satisfied whenever possible;
(c) "gaps" in the time schedule of teachers should be avoided, that is: periods of no activity between two lesson periods.

3.1 Solution Representation

A timetable is represented as a matrix $Q_{t \times p}$, in such a way that each row represents the complete weekly timetable for a given teacher. As such, the value $q_{ik} \in \{0, 1, \cdots, c\}$, indicates the class for which the teacher i is teaching during period k ($q_{ik} \in \{1, \cdots, c\}$), or if the teacher is available for allocation ($q_{ik} = 0$). The advantage of this representation is that it eliminates the possibility for the occurrence of conflicts for teachers. The occurrence of conflicts in classes happens when in a given period k more than one teacher is allocated to that class. Allocations are only allowed in periods with teacher availability. A partial sample of a timetable with 5 teachers can be found in Figure 1, with value "X" indicating unavailabilities of teachers.

3.2 Objective Function

In order to treat CTTP as an optimization problem, it is necessary to define an objective function that determines the degree of infeasibility and dissatisfaction of requirements; that is, pretends to generate feasible solutions with a minimal number of unsatisfied requisites. Thus, a timetable Q is evaluated with the following objective function, which should be minimized:

$$f(Q) = \omega \times f_1(Q) + \delta \times f_2(Q) + \rho \times f_3(Q) \tag{1}$$

where f_1 counts, for each period k, the number of times that more than one teacher teaches the same class in period k and the number of times that a class has no activity in k. The f_2 portion measures the number of allocations that disregard the daily limits of lessons (constraint (d)). As such, a timetable can only be considered feasible if $f_1(Q) = f_2(Q) = 0$. The importance of the costs involved defines a hierarchy so that: $\omega > \delta \gg \rho$. The f_3 component in the objective function measures the dissatisfaction of personal requests from teachers, namely: double lessons, non-existence of "gaps" and timetable compactness, as follows:

$$f_3(Q) = \sum_{i=1}^{t} (\alpha_i \times g_i + \beta_i \times v_i + \gamma_i \times l_i) \tag{2}$$

where α_i, β_i, and γ_i are weights that reflect, respectively, the relative importance of the number of "gaps" g_i , the number of week days v_i each teacher is allocated for teaching, and the non-negative difference l_i between the minimum required number of double lessons and the effective number of double lessons in the current agenda for teacher i.

4. TABU SEARCH FOR THE CLASS/TEACHER TIMETABLING PROBLEM

Tabu Search (TS) is an iterative method for solving optimization problems. It explicitly makes use of memory structures to guide a hill-descending heuristic to continue exploration without being confused by the absence of improvement movements. This technique was independently proposed by Glover [Glover 1986] and Hansen [Hansen 1986]. For a detailed description of TS, the reader is referred to [Glover and Laguna 1997]. This section presents a brief explanation of TS principles. They are followed by specifications of the customized TS implementation proposed in this paper.

Starting from an initial solution x, the method systematically explores a subset $\mathcal{V}(x)$ of the neighborhood $\mathcal{N}(x)$ and selects the best admissible movement m, so that the application of m in the current solution x (denoted by $x \oplus m$) produces the new current solution $x' \in \mathcal{V}(x)$. Movements that deteriorate the cost function are also permitted. Thus, to try to avoid cycling, a mechanism called short term memory is employed. The objective of short term memory is to try to forbid movements toward already visited solutions, which is usually achieved by the prohibition of undoing the last performed movements. These movements are stored in a tabu list and remain forbidden (with tabu status), for a given number of iterations, called *tabu tenure*. Since this strategy can be too restrictive, so as not to disregard high quality solutions, movements with tabu status can be accepted if the new solution produced satisfies an *aspiration criterion*. Also, intensification and diversification procedures can be used. These procedures, respectively, aim to deeply investigate promising regions of the search space and to ensure that no region of the search space remains neglected. Following is a description of the constructive algorithm and the customized TS implementation proposed in this paper.

```
procedure GenerateTimetable($\alpha, R, A, P$)
 1: $\zeta_{ij} \leftarrow r_{ij}$ $(i \in T, j \in C)$;
 2: $V_i \leftarrow A_i$ $(i \in T)$;    $W_j = P$ $(j \in C)$;
 3: repeat
 4:     $\theta_{ij} = \frac{\zeta_{ij}}{|V_i \cap W_j|+1}$;
 5:     $RCL = \{(i,j) \mid \theta_{ij} \geq \overline{\theta} - (\overline{\theta} - \underline{\theta}) \times \alpha\}$;
 6:     Randomly select $(d,e)$, such that $(d,e) \in RCL$;
 7:     $F \leftarrow V_d \cap W_e$
 8:     if $F = \emptyset$ then
 9:         $F \leftarrow V_d$;
10:     end if
11:     Associate probabilities to periods and randomly select $f \in F$;
12:     $Q_{df} \leftarrow e$;
13: until $\exists \zeta_{ij} > 0$ $(i \in T, j \in C)$;
14: return $Q$;
end GenerateTimetable.
```

Fig. 2. Pseudo-code for GenerateTimetable

4.1 Constructive Algorithm

The constructive algorithm basically consists of a greedy randomized constructive procedure [Resende and Ribeiro 2003]. Although in other works the option for a randomized construction is to provide diversification, through multiple re-initializations, in our implementation the only purpose is to have control of the randomization degree of initial solution. The construction procedure (Figure 2) is somewhat similar to the human way of building timetables. To build a solution, step-by-step, the principle of allocating first the *most urgent* lessons in the *most appropriate* periods is used. Receiving problem data A, R and P (section 3), the algorithm computes, at each iteration, the urgency degree θ_{ij} of allocating a lesson from teacher i for class j considering available periods V_i from teacher i, available periods W_j from class j and the number of unscheduled lessons ζ_{ij} of teacher i for class j, as follows: $\theta_{ij} = \frac{\zeta_{ij}}{|V_i \cap W_j|+1}$. The algorithm then builds a restricted candidate list (RCL) with ordered pairs (i,j) with highest urgency degrees, such that $RCL = \{(i,j) \mid \theta_{ij} \geq \overline{\theta} - (\overline{\theta} - \underline{\theta}) \times \alpha\}$, where $\overline{\theta} = \max\{\theta_{ij} \mid i \in T,\ j \in C\}$ and $\underline{\theta} = \min\{\theta_{ij} \mid i \in T,\ j \in C\}$. At each iteration, one lesson from teacher i and class j, such that $(i,j) \in RCL$, is randomly selected for allocation. The α parameter $(0 \leq \alpha \leq 1)$ allows tuning the randomization degree of the algorithm, varying from the pure greedy lesson selection ($\alpha = 0$) to a completely random ($\alpha = 1$) selection of teacher and class for allocation.

The selection of the period in which the selected lesson will be allocated is done in free periods of teachers, trying to prevent clashes in classes timetables (this constraint is violated whenever $W_j \cap V_i = \emptyset$). To provide another level of diversity in the initial solution, the selection of period for allocation is also probabilistic, in a way that periods with low availability of teachers will have an exponentially bigger probability of being chosen [Bresina 1996].

At each iteration, the number of unscheduled lessons, availabilities of teachers and classes and urgency degrees are recomputed. The process continues until no more unscheduled lessons are found (i.e.: $\zeta_{ij} = 0, i \in T, j \in C$).

```
procedure ImproveTimetable($Q$, $divActivation$, $iterationsDiv$, $c_{tenure}$, $\varphi$)
1: $Q^* = Q$; $TabuList = \emptyset$; $noImprovementIterations = 0$; $iteration = 0$;
2: $initializeLongTermMemory()$;
3: repeat
4:    $\Delta = \infty$; $iteration++$; $bestMov = randomMovement()$;
5:    for all movement $m$ such that $(Q \oplus m) \in \mathcal{N}(Q)$ do
6:       $penalty = 0$;
7:       if ($noImprovementIterations$ mod $divActivation < iterationsDiv$)
              and ($iteration \geq divActivation$) then
8:          $penalty = computePenalty(m)$;
9:       end if
10:      $\Delta' = f(Q \oplus m) - f(Q)$;
11:      if ( ($\Delta' + penalty < \Delta$) and ($m \notin TabuList$) )
             or ( ($f(Q \oplus m) < f(Q^*)$) and ($\Delta' < \Delta$) ) then
12:         $bestMov = m$;
13:         $\Delta = \Delta'$;
14:         if ($f(Q \oplus m) \geq f(Q^*)$) then $\Delta = \Delta + penalty$;
15:      end if
16:   end for
17:   $updateLongTermMemory(bestMov, Q)$;
18:   $Q = Q \oplus bestMov$;
19:   $tabuTenure(bestMov) = random(\lfloor c_{tenure} - \varphi \times c_{tenure} \rfloor, \lceil c_{tenure} + \varphi \times c_{tenure} \rceil)$;
20:   $updateTabuList(bestMov, iteration)$;
21:   if ($f(Q) < f(Q^*)$) then
22:      $Q^* = Q$; $noImprovementIterations = 0$;
23:      $initializeLongTermMemory()$;
24:   else
25:      $noImprovementIterations++$;
26:   end if
27: until termination criterion reached;
28: return $Q^*$;
end ImproveTimetable.
```

Fig. 3. Pseudo-code for tabu search algorithm to the class/teacher timetabling problem

4.2 Tabu Search Components

The TS procedure (Figure 3) starts from the initial timetable Q provided by the constructive algorithm and, at each iteration, fully explores the neighborhood $\mathcal{N}(Q)$ (in this implementation $\mathcal{V}(Q) = \mathcal{N}(Q)$) to select the next movement m. The movement definition used here is the same as in [Schaerf 1999], and involves the swapping of two values in the timetable of a teacher $i \in T$, which can be defined as the triplet $\langle i, p_1, p_2 \rangle$, such that $q_{ip_1} \neq q_{ip_2}$, $p_1 < p_2$ and $p_1, p_2 \in \{1, \cdots, p\}$. Clearly, any timetable can be reached through a sequence of these movements that is, at most, the number of lessons in the requirements matrix. The time complexity of exploring $\mathcal{N}(x)$ is $O(t \cdot p^2 \cdot EV)$ where EV is the cost of evaluating each neighbor solution, which can be efficiently done by recomputing only costs related to teacher i and conflicts in periods p_1 and p_2.

Once a movement m is selected, it will be kept in the tabu list during the next $tabuTenure(m)$ iterations. In order to hinder the occurrence of cycling, $tabuTenure(m)$ is not a fixed value, but is randomly selected from values close to a central value (c_{tenure} input parameter). The allowable deviation from this value is defined by the φ input parameter ($\varphi \in [0, 1]$), such that it will determine the range of possible values for tabu tenure (line 19). Insertions and removals in tabu list can be made at every iteration (line 20). The aspiration criterion defined is that the movement will lose its tabu status if its application produces the best solution found so far (line 11).

Since short term memory is not enough to prevent the search process from becoming trapped in certain regions of the search space, some diversification strategy is necessary. In the proposed method, long term memory is used to guide the diversification procedure. The motivation to employ a memory guided diversification procedure instead of random re-start is twofold: firstly, information loss incurred from random re-start is avoided and secondly, the use of memory to guide the diversification process, hopefully, diminishes the risk of revisiting the same region of the search space.

Two types of long term memory are proposed. The first type involves the storage of transition measures, counting the frequency of movements involving each teacher and class. The second type involves the storage of residency measures counting the number of times in which each lesson has occupied a given period. Every time a movement is done, long term memory information is updated (line 17), and every time the best solution is updated, long term memory is cleared (line 23).

While the diversification strategy is active, long term memory information is used to guide the selection of movements, so that movements in slightly modified timetables and/or movements which make unusual allocations are encouraged. This is done through the incorporation of penalties in the evaluation of movements (line 8). In the following paragraphs a description of the proposed long term memories and how they are used to compute penalties in the diversification strategy is presented.

4.2.1 *Transition based long term memory.* In this type of memory, transition measures are stored in a matrix $Z_{t\times c}$, counting how many movements z_{ij} were done involving teacher i and class j. Using these values, transition ratios are computed. Let $\overline{z} = \max\{z_{ij} \mid i \in T,\ j \in C\}$, the transition ratio ϵ_{ij} for teacher i and class j is:

$$\epsilon_{ij} = \frac{z_{ij}}{\overline{z}} \tag{3}$$

Since a movement can involve two lesson periods, or a lesson period and a free period, the penalty used in the diversification strategy $\psi_{ia_1a_2}$ associated with a movement in the timetable of teacher i, in periods p_1 and p_2 with allocations $a_1 = q_{ip_1}$ and $a_2 = q_{ip_2}$, respectively, considering the cost of the current solution $f(Q)$ is:

$$\psi_{ia_1a_2} = \begin{cases} \epsilon_{ia_1} \times f(Q) & \text{if } a_1 \neq 0 \text{ and } a_2 = 0 \\ \epsilon_{ia_2} \times f(Q) & \text{if } a_1 = 0 \text{ and } a_2 \neq 0 \\ (\epsilon_{ia_1} + \epsilon_{ia_2})/2 \times f(Q) & \text{if } a_1 \neq 0 \text{ and } a_2 \neq 0 \end{cases}$$

Initialization of Z requires $O(t \cdot c)$ operations. Updates and lookups can be done in $O(1)$.

4.2.2 *Residence based long term memory.* In this type of memory, residence measures are stored for each lesson, in a $Y_{t\times c\times u\times p}$ matrix ($u = max\{r_{ij} \mid i \in T, j \in C\}$), where y_{ijmk} expresses how many iterations the m^{th} lesson of teacher i on class j occupied period k. Although it is a fourth-dimensional matrix, this is a very sparse matrix, in a way that efficient implementations make its use practical for problems considered in this paper. To compute the residence ratio η_{ijmk} of the m^{th} lesson of teacher i and class j on period k, the maximum value of y_{ijmk} $(i \in T, j \in C, m \in \{1, 2, \cdots, u\}, j \in P)$ $\overline{y}$ is considered, as follows:

$$\eta_{ijmk} = \frac{y_{ijmk}}{\overline{y}} \tag{4}$$

Thus, the penalty μ_{ijmk} for allocating the m^{th} lesson of teacher i and class j on period k is:

$$\mu_{ijmk} = \eta_{ijmk} \times f(Q) \tag{5}$$

By using hash tables, lookups for the residence ratio can be done in $O(1)$ average time. Update has $O(t \cdot p)$ time complexity.

For movements which involve two allocations in a timetable of a given teacher the penalty will be the average penalty of the involved lessons.

The diversification strategy is applied whenever signals that regional entrenchment may be in action are detected. In this case, the number of non-improvement iterations is evaluated before starting the diversification strategy (line 7). The number of non-improvement iterations necessary to start the diversification process (*divActivation*) and the number of iterations that the process will remain active (*iterationsDiv*) are input parameters. The process is cyclic and restarts whenever a multiple of *divActivation* non-improvement iterations is reached. Movements performed in this phase can be viewed as *influential movements* [Glover and Laguna 1997], since these movements try to modify the solution structure in a influential (non-random) manner. The function *computePenalty* (line 8) can use one of the proposed long term memory based penalties. In the following sections the tabu search implementation with transition based long term memory will be referred as `TST`, while the implementation with residence based long term memory will be referred as `TSR`. Another implementation, which maintains both types of long term memory will be referred as `TSTR`. In `TSTR`, penalties computed using transition based long term memory and residence based long term memory are summed and used in the diversification strategy.

For comparison purposes, an implementation without any diversification strategy (`TS`), also will be considered in next sections.

5. COMPUTATIONAL EXPERIMENTS AND DISCUSSION

Experiments were done in the set of instances originated from [Souza et al. 2003], and the data referred to Brazilian high schools, with 25 lesson periods per week for each class, in different shifts. In Table I some of the characteristics of the instances can be verified, such as dimension and sparseness ratio (sr), which can be computed considering the total number of lessons ($\#lessons$) and the total number of unavailable periods ($\#u$): $sr = \frac{t \times p - (\#lessons + \#u)}{t \times p}$. Lower sparseness values indicate more restrictive problems and, likewise, problems in which it is more difficult to find feasible timetables.

Three objectives guided the selection of computational experiments to be included in this work: firstly, to search for the best parameters and modules com-

Table I. Characteristics of problem instances

Instance	*Teachers*	*Classes*	*Total Lessons*	*Double Lessons*	*Sparseness Ratio*(sr)
1	8	3	75	21	0.43
2	14	6	150	29	0.50
3	16	8	200	4	0.30
4	23	12	300	66	0.18
5	31	13	325	71	0.58
6	30	14	350	63	0.52
7	33	20	500	84	0.39

Table II. Average distance from best known solution - `TS`

	Central tabu tenure				
Instance	15	20	25	30	*Average*
1	5.13	2.32	0.78	1.28	2.38
2	3.47	2.21	1.15	1.72	2.14
3	7.94	6.68	2.48	0.92	4.50
4	0.97	0.96	0.82	0.24	0.75
5	4.83	6.32	1.96	0.45	3.39
6	3.45	2.14	0.94	0.30	1.71
7	1.75	1.65	0.54	0.82	1.19
Average	3.93	3.18	1.24	0.82	2.29

Table III. Average distance from best known solution - `TST`

	Central tabu tenure				
Instance	15	20	25	30	*Average*
1	0.35	0.30	0.15	0.10	0.22
2	0.00	0.12	0.52	0.38	0.25
3	0.46	1.03	0.27	0.00	0.44
4	0.36	0.27	0.27	0.31	0.30
5	0.22	0.21	0.24	0.37	0.26
6	0.19	0.27	0.45	0.45	0.34
7	0.00	0.19	0.43	0.55	0.29
Average	0.22	0.34	0.33	0.31	0.30

position (which diversification strategy gives better results), secondly, verify how the proposed tabu search heuristic compares to the previously proposed `GTS-II` algorithm, and thirdly verify how good are the provided solutions, considering its practical application.

The algorithms were coded in C++ and the implementation of `GTS-II` was the same presented in [Souza et al. 2003]. The compiler used was GCC 3.2.3 using flag `-O2`. Experiments were performed in micro-computers with AMD Athlon XP 1533 MHz processors, 512 megabytes of RAM, running the Linux operating system.

The weights in the objective function were defined as in [Souza et al. 2003]: $\omega = 100$, $\delta = 30$, $\rho = 1$, $\alpha_i = 3$, $\beta_i = 9$ and $\gamma_i = 1$, $\forall i = 1, \cdots, t$.

Initially, experiments to verify which is the best parameter configuration for the proposed algorithms were done (parameters for `GTS-II` were the same used in [Souza et al. 2003]). Average results of 10 independent executions (different random seeds) on each instance for different central tabu tenure values (c_{tenure})

Table IV. Average distance from best known solution - `TSR`

	Central tabu tenure				
Instance	15	20	25	30	*Average*
1	0.59	0.15	0.00	0.20	0.23
2	0.61	0.32	0.20	0.41	0.39
3	0.14	0.60	0.76	1.17	0.66
4	0.24	0.21	0.34	0.37	0.29
5	0.30	0.00	0.35	0.31	0.24
6	0.00	0.39	0.39	0.41	0.30
7	0.45	0.63	0.38	0.72	0.55
Average	0.33	0.33	0.34	0.51	0.38

Table V. Average distance from best know solution - `TSTR`

	Central tabu tenure				
Instance	15	20	25	30	*Average*
1	0.25	0.10	0.15	0.25	0.19
2	0.26	0.44	0.61	0.61	0.48
3	0.73	0.30	0.53	0.53	0.52
4	0.00	0.15	0.19	0.43	0.19
5	0.05	0.09	0.56	0.55	0.31
6	0.08	0.19	0.53	0.59	0.35
7	0.04	0.20	0.36	0.44	0.26
Average	0.20	0.21	0.42	0.49	0.33

and instances were computed (other parameters remain fixed: $\alpha = 0.1$, $\varphi = 0.1$, $divActivation = 500$ and $iterationsDiv = 10$). Executions had fixed time limits, as proposed in [Souza et al. 2003], which are for instances $\{1, \cdots, 7\}$, respectively: $\{90,$ $280, 380, 870, 1930, 1650, 2650\}$ seconds. In Tables II, III, IV and V, the average distance of the cost of generated solutions from the best known solution is shown, for strategies with and without the diversification component. As it can be seen, for `TS` (without diversification strategy), better results were obtained with the highest c_{tenure} values. Nevertheless, implementations with the proposed diversification strategies obtained better results, with any c_{tenure} value, than the simple `TS`. While, in average, `TST` performed better than `TSR`, the best results were obtained in the implementation which considers both types of long term memory, using low c_{tenure} values, since `TSTR` with $c_{tenure} = 15$ generated solutions, in average, only 0.20% distant from best known solution. From now on, results of the proposed algorithms consider experiments with parameters which produced better average results (i.e., for c_{tenure} : 30 for `TS` and 15 for `TST`, `TSR` and `TSTR`).

A different view of the results of the previously described experiment is presented in Table VI. Average solution costs generated by proposed algorithms are compared to average results of `GTS-II` within the same time limits. Best results are shown in bold.

Results in Table VI demonstrate that although only minor differences can be observed among implementations that use different penalty functions in the diversification strategy, versions using informed diversification strategies perform significantly better than `GTS-II` and `TS`.

In order to evaluate the quality of the solutions obtained by the proposed method,

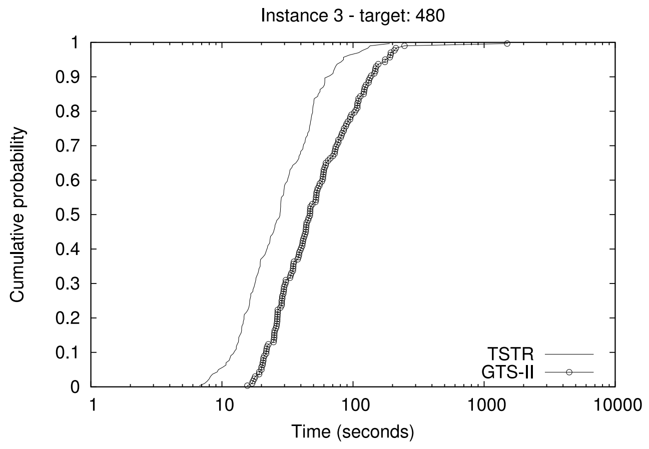
Table VI. Average results, runs with fixed time limit

Instance	GTS-II	TS	TST	TSR	TSTR
1	204.80	205.30	203.40	**203.00**	203.20
2	350.10	349.20	**343.30**	344.40	344.20
3	455.70	440.90	**438.90**	439.50	440.10
4	686.30	670.50	671.30	670.30	**668.90**
5	796.30	782.70	780.90	**779.20**	779.60
6	799.10	781.50	780.70	782.20	**779.80**
7	1,076.20	1,063.80	**1,055.20**	1,061.90	1,055.60

Table VII. Average costs of objective function components obtained by the constructive algorithm and at the end of the tabu search heuristic

			$f_3(Q^*)$		
Instance	$f_1(Q^*)$	$f_2(Q^*)$	$\sum_{i=1}^{t} l_i(\%l)$	$\sum_{i=1}^{t} g_i(\%g)$	cr
			Constructive Algorithm		
1	0.0	0.5	15.1(71.5)	17.2(22.9)	1.6
2	0.0	0.0	24.3(83.8)	24.8(16.5)	1.3
3	0.3	2.5	2.0(50.0)	31.2(15.6)	1.4
4	4.3	0.9	35.5(53.8)	21.0(7.0)	1.2
5	0.0	0.2	54.1(76.1)	46.4(14.3)	1.5
6	0.2	0.0	53.7(85.2)	53.4(15.3)	1.4
7	0.5	0.2	69.6(82.9)	74.1(14.8)	1.3
			At the end of the TSTR heuristic		
1	0.0	0.0	1.9(9.0)	4.1(5.5)	1.2
2	0.0	0.0	7.3(25.2)	1.3(0.9)	1.0
3	0.0	0.0	0.4(10.0)	5.4(2.7)	1.1
4	0.0	0.0	19.4(29.4)	3.8(1.3)	1.0
5	0.0	0.0	13.7(19.3)	3.5(1.1)	1.1
6	0.0	0.0	15.4(24.4)	8.8(2.5)	1.0
7	0.0	0.0	23.0(27.4)	10.6(2.1)	1.0

taking into account its practical application, and to verify how significant is the improvement of TSTR over the solution received from the constructive algorithm, Table VII presents the average costs involved in each objective function component, for the solution provided by the constructive algorithm and for the improved solution from TSTR. The three last columns are related to the f_3 component of the objective function (section 3.2) where $\%l$ is the percentage of unsatisfied double lessons considering the number of double lessons requests and $\%g$ represents the percentage of "gaps" in the timetable of teachers considering the total number of lessons. cr measures the compactness ratio of timetable of teachers. To compute cr, the summation of the actual number of days ad that each teacher must attend to some lessons in the school and the lower bound for this value $\underline{ad}$ are used. The $\underline{ad}$ value considers the minimum number of days $md_i = \left\lceil \frac{\sum_{j=1}^{c} r_{ij}}{h} \right\rceil$ that each teacher i must attend some lectures in the school, such that $\underline{ad} = \sum_{i=1}^{t} md_i$. This way, $cr = ad/\underline{ad}$. Values close to one indicate that the timetable is as compact as it can be. As can be seen in Table VII, the solution provided by the constructive algorithm usually contains some type of infeasibility. These problems were always solved by the TSTR algorithm, in a way that no infeasible timetable was produced. Regarding the preferences of teachers, the timetable compactness, which has the



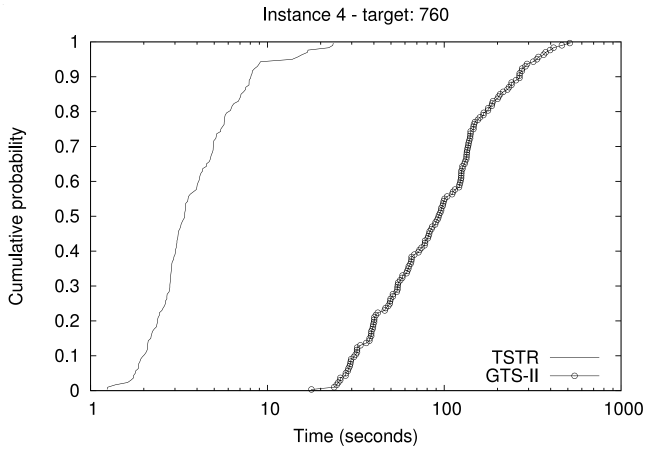


Fig. 4. Empirical probability distribution of finding target value in function of time for instances 3 and 4

highest weight in the f_3 component of the objective function, it can be seen that in most cases the optimal value was reached ($cr = 1$). Also, small percentage values were obtained for “gaps” and unsatisfied double lessons.

Table VIII. Time (in seconds) for 25%, 50% and 75% of runs achieve the target solution values.

	GTS-II			TSTR		
Instance	25%	50%	75%	25%	50%	75%
1	7.64	9.57	12.15	2.13	3.36	6.39
2	21.39	26.57	34.68	9.03	13.48	19.71
3	28.57	46.84	85.41	16.29	27.66	46.47
4	49.22	92.57	146.50	2.65	3.40	5.45
5	47.79	62.85	102.20	27.63	37.85	54.51
6	35.81	48.00	72.12	25.20	33.97	44.38
7	92.41	150.72	287.48	89.57	118.82	155.72

In another set of experiments, the objective was to verify the empirical probability distribution of reaching a given sub-optimal target value (i.e., find a solution with cost at least as good as the target value) in function of time in different instances. The sub-optimal values were chosen in a way that the slowest algorithm could terminate in a reasonable amount of time. In these experiments, TSTR and GTS-II were evaluated and the execution times of 150 independent runs for each instance were computed. The experiment design follows the proposal of [Aiex et al. 2002]. The results of each algorithm were plotted associating with the i^{th} smallest running time t_i a probability $p_i = (i - \frac{1}{2})/150$, which generates points $z_i = (t_i, p_i)$, for $i = 1, \cdots, 150$. The results shown that TSTR achieves high probability values ($\geq 50\%$) of reaching the target values in significantly smaller times than GTS-II, for all instances. Representative results are presented in Figures 4 and 5.

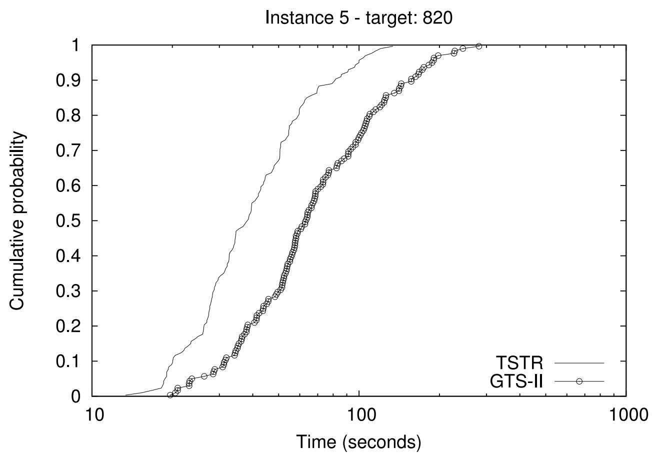
This difference is enhanced mainly in instance 4, which presents a very low sparseness ratio. This result may be related to the fact that the "Intraclasses-Interclasses" procedure of GTS-II works with movements that use free periods, which are hard to find in this instance. Another analysis, taking into account all test instances, shows that at the time when 95% of TSTR runs have achieved the target value, in average, only 64% of GTS-II runs have achieved the target value. Considering the time when 50% of TSTR runs have achieved the target value, only 11%, in average, of GTS-II runs have achieved the target value. Table VIII presents the execution times needed by GTS-II and TSTR to achieve different probabilities of reaching the target values.

6. CONCLUDING REMARKS

This paper presented a new tabu search heuristic to solve the class/teacher timetabling problem. Experiments on real world instances showed that the proposed method outperforms significantly a previously developed hybrid tabu search algorithm, and it has the advantage of a simpler design.

Contributions of this paper include the empirical verification that although informed diversification strategies are not commonly employed in tabu search implementations for the class/teacher timetabling problem, their incorporation can significantly improve the robustness of the method. The proposed method not only produced better solutions for all test instances but also performed faster than a hybrid tabu search approach.

Although in the proposed algorithm long term memory was used to guide diversification procedures, intensification strategies which use this type of information can



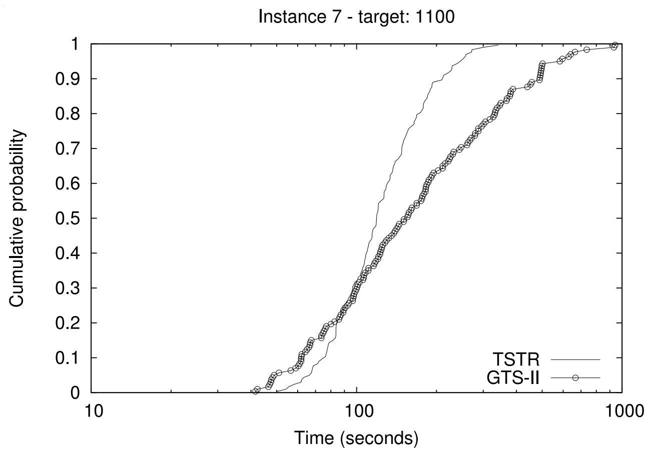


Fig. 5. Empirical probability distribution of finding target value in function of time for instances 5 and 7

be formulated, and their application is worthy of receiving further investigation.

Other interesting enhancement to the algorithm could be the combination of the "Intraclasses-Interclasses" procedure with an informed diversification strategy, which could lead to even better results.

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